

# A k-distribution technique for radiative transfer simulation in inhomogeneous atmosphere: 1. FKDM, fast k-distribution model for the longwave

Boris A. Fomin

CPTEC/INPE, Cachoeira Paulista, Sao Paulo, Brazil

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[1] A new technique for developing k-distributions is described. This new technique requires almost half of the computational effort yet is at least as accurate as other published k-distribution methods. Its novelty consists in the use of real atmospheric flux calculations to guide both the position of spectral bands and the k terms within each band; previous methods have not used such a guiding principle. Wave number subintervals which have similar atmospheric absorption behavior are chosen, then a representative absorption coefficient for such a wave number interval is set to the value which best fit the results of line-by-line calculations of fluxes and heating rates. This method of choosing one absorption coefficient to represent a large wave number subinterval contrasts with other published methods and is responsible in large part for the improved computational efficiency. It is worth noting that this new technique works as well in the stratosphere as the troposphere, so it can be applied to the processing of satellite data retrievals as well as in weather and climate forecasting. An example of the application of the new technique to the longwave part of the spectrum is presented. A fast k-distribution model (FKDM) suitable for use in weather and climate prediction has been created using 23 k-distribution terms, which is nearly half as many as other k-distribution models. The molecular species represented in the model are H<sub>2</sub>O, CO<sub>2</sub>, O<sub>3</sub>, N<sub>2</sub>O, CH<sub>4</sub>, and CFC-11,12,113. FKDM has been developed and validated using a fast line-by-line model (FLBLM). Validations have covered the tropical, midlatitude summer, midlatitude winter, subarctic summer, subarctic winter standard atmospheres, four atmospheres from the Spectral Radiance Experiment campaign, and one case of real tropical atmosphere. It has been found that the FKDM cooling rate accuracy is as follows: 0.15 and 0.2 K day<sup>-1</sup> in troposphere for standard and real atmospheres, respectively, and 0.9 K day<sup>-1</sup> in all the cases at altitudes below 70 km. Upward and downward flux errors are below 4 W m<sup>-2</sup> (usually 1–2 W m<sup>-2</sup>) in every case. **INDEX TERMS:** 3309 Meteorology and Atmospheric Dynamics: Climatology (1620); 3319 Meteorology and Atmospheric Dynamics: General circulation; 3337 Meteorology and Atmospheric Dynamics: Numerical modeling and data assimilation; 3359 Meteorology and Atmospheric Dynamics: Radiative processes; 3367 Meteorology and Atmospheric Dynamics: Theoretical modeling; **KEYWORDS:** radiation code, longwave, climate model

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## 1. Introduction

[2] At present, huge computational resources are required for the calculation of radiative transfer in the atmosphere, for weather and climate prediction, and the processing of radiance data retrieved by satellites. A new fast radiation code is described, based on the k-distribution method of parametrizing atmospheric absorption [Ambartsumian, 1936; Lacis and Hansen, 1974; Wiscombe and Evans, 1977; Lacis and Oinas, 1991]. It is suggested that this new technique finds the shortest k-distribution series achievable in practice. Since the computational time is proportional to the number of terms in the k-distribution

series, this technique is particularly useful for fast radiation codes.

[3] This new technique is more effective than the established correlated-k distribution method [e.g., Lacis and Oinas, 1991] because of the different use of the most accurate models of radiative transfer, known as line-by-line models. The correlated-k distribution method uses line-by-line calculations only to get absorption coefficient spectra at several atmospheric levels. Then a procedure which sorts the absorption coefficients by value creates a new, artificial spectra, on a changed wave number grid, which in turn permits radiative transfer calculations to be done by means of standard quadrature (e.g., Gauss-Legendre). Nonstandard quadrature has also been done [e.g., Cusack et al., 1999] in an attempt to reduce the number of quadrature points

because this defines the k-distribution terms and thus the speed of execution of the code. The values of the artificial spectra at atmospheric levels at the quadrature point of the changed wave number grid give a vertical profile of the representative absorption coefficient for each term. Line-by-line calculations of fluxes and cooling rates are used for assessment of the parametrization and in particular the choice of quadrature points. It should be stressed that no standard quadrature can ensure the minimal number of terms because it has the maximum efficiency in case of defined integrands only. In contrast, this new technique makes use of line-by-line calculations twice: (1) for calculation of an absorption spectrum at a single level of an atmospheric model to get wave number subsets where absorption coefficients feature similar behavior (neither the sorting procedure nor the quadrature method used in the correlated-k distribution technique are used here) and (2) for calculations of cooling rates and fluxes for each wave number subset in real inhomogeneous atmospheres. A number of wave number subsets give a corresponding number of k-distribution terms and a representative absorption coefficient for each term is defined from inverting the above line-by-line flux calculations. The efficiency of this new technique is manifested most vividly in cooling rate simulation in stratosphere and upper atmosphere.

[4] I am not sure it is fair to say that the correlated-k distribution techniques have no information on areas of strong and weak absorption in spectral regions, as the method of sorting effectively gives some k terms which are strong and some which are weak. I think it might be fairer to say that the new method has a more direct representation of overlapping gases and is less susceptible to errors.

[5] This new technique requires intensive line-by-line radiative calculations, more so than with other k distribution methods, but the recent advanced in computer hardware and the use of a fast line-by-line model (FLBLM) [Fomin, 1994; Fomin *et al.*, 2004] make this approach much more feasible today.

[6] Some aspects of the technique have been recently discussed in brief in the work of Fomin [2003], where only several bands of H<sub>2</sub>O and CO<sub>2</sub> were considered. In this work we will give a detailed description of this technique and of the first completed version of the k-distribution model intended mainly for longwave radiation simulation in climate models. In this version we considered absorption by H<sub>2</sub>O, CO<sub>2</sub>, O<sub>3</sub>, N<sub>2</sub>O, CH<sub>4</sub>, CFC-11, CFC-12, and CFC-113 that needs only 23 k-distribution terms.

[7] Section 2 of the paper presents a new technique to generate k-distribution. Section 3 describes the author's fast k-distribution model (FKDM) in detail. Section 4 is devoted to the models validation, where validation of both the FLBLM and the FKDM models using a set of real atmospheres is considered in subsection 4.1. Subsection 4.2 describes validation of the FKDM sensitivity to variations of CO<sub>2</sub>, N<sub>2</sub>O, CH<sub>4</sub>, and halocarbons concentrations. Finally, section 5 is a summary.

## 2. Technique to Generate k-Distributions

[8] As is well known, if absorption is independent of a wave number within a certain spectral interval, then upward

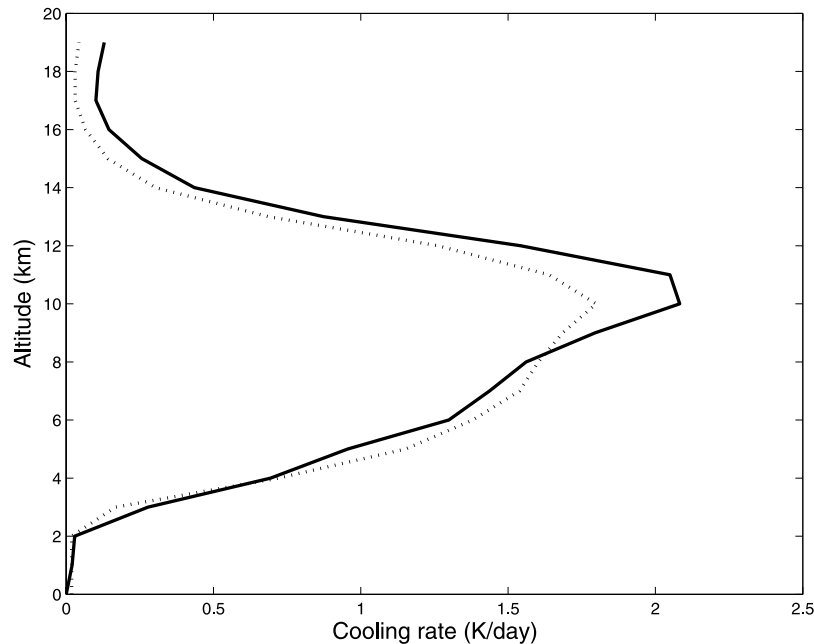
$F_U(Z)$  and downward  $F_D(Z)$  fluxes and consequently cooling rates  $Q(Z)$  in horizontally homogeneous atmosphere can be calculated at any altitude  $Z$  using general formulae for monochromatic radiation [e.g., Goody and Yung, 1989]. Of course, a vertical profile of volume absorption coefficient  $K(Z)$  must be known for this calculation. If it is unknown, but we have one of the above three profiles, (e.g.,  $F_U(Z)$ ), then it is possible to define  $K(Z)$  by solving an inverse problem. Then the other two profiles ( $F_D(Z)$  and  $Q(Z)$  in our example) can be calculated in the usual way.

[9] An algorithm for the solution of the inverse problem is rather simple. For example, if a downward flux,  $F_D(Z)$ , is used, calculations start with the highest layer of the atmospheric model. At the top of this layer there is a boundary condition,  $F_D(Z_{\max}) = 0$ , and the optical depth equals zero. Using this condition, the optical thickness of this layer (optical depth at the lower boundary) can be defined with a simple iterative procedure (e.g., bisection) so that it gives a downward flux value at the lower boundary of the layer, which agrees with that from a line-by-line model. Thus we get the optical depth and the downward flux at the upper bound of the underlying layer. Then the same iterative procedure provides the means for finding the optical depth and the downward flux at the lower boundary of this layer (which is the upper bound of the next layer). By repeating this procedure layer after layer, the optical depth at each level of the atmospheric model can be defined. This is a straightforward procedure to define, first, volume absorption coefficients  $K$  (km<sup>-1</sup>) and, second, using the concentration of model gas molecules, cross sections  $\bar{K}$  (cm<sup>2</sup> molecule<sup>-1</sup>) at each level.

[10] If the absorption does depend upon wave number, then the above method will give corresponding errors. Figure 1 shows the cooling rate profile calculated by FLBLM due to H<sub>2</sub>O in the spectral region 50–550 cm<sup>-1</sup> for the Air Force Geophysical Laboratory (AFGL) Tropical atmosphere. Also shown in Figure 1 is the cooling rate profile estimated using the  $K(Z)$  obtained from FLBLM's  $F_D(Z)$ . Since absorption does depend on wave number in this spectral interval, the two methods of calculation give visible differences for cooling rates as a result of a good agreement for downward fluxes and a disagreement for upward fluxes. It is worth noting that this simple one-term approximation achieves greater accuracy for the narrower spectral region of 50–250 cm<sup>-1</sup> and that the same profile of  $K(Z)$  produces similar errors (less than 0.1 K/day) for other atmospheres [Fomin, 2003].

[11] The underlying idea of this technique is rooted in the analysis of the discrepancies shown in Figure 1. They can be easily explained by weak and strong absorption mixing in the single k-distribution term. It is clear that weak and strong absorption should be taken into account separately in different terms to improve the accuracy of the approximation. The separation procedure can be viewed upon as adequate if a single absorption profile in each term simulates both fluxes and cooling rates with the required accuracy. In this sense, the correlated-k distribution method is appropriate. Here another technique will be described in detail, which uses comparison line-by-line and approximate calculations directly. It consists of several stages.

[12] 1. A set of volume absorption coefficients  $k_i(Z)$  (separate for each essential atmospheric gas) must be



**Figure 1.** Cooling rates calculated with line-by-line (solid line) and approximate one-term models (dotted line). Tropical atmosphere and absorption by  $\text{H}_2\text{O}$  in the  $50\text{--}550\text{ cm}^{-1}$  spectral region are assumed.

calculated by any line-by-line model for all the horizontal levels  $Z$  of the chosen atmospheric model at each  $i$ -th point of the wave number grid.

[13] 2. A certain level  $Z^*$  should be selected. Absorption coefficients calculated at this level will be used for selection of the wave number points into the point subset  $U^1$ , where absorption is approximately the same (and weak in comparison with absorption of other wave number subsets). To this aim, a threshold  $S^1$  should be guessed. The selection rule is very simple: the  $i$ -th point should be included into the point subset  $U^1$  if absorption coefficient  $k_i$  is less than  $S^1$  ( $i \in U^1$  if  $k_i < S^1$ ).

[14] 3. The three profiles of both fluxes and the cooling rate should be calculated (using line-by-line technique) only for the wave number points included into subset  $S^1$ .

[15] 4. Then only one of them (e.g.,  $F_D(Z)$ ) should be used to calculate an absorption coefficients profile  $K^1(Z)$  in the above-discussed manner (by solving the inverse problem).

[16] 5. Profile  $K^1(Z)$  should be substituted in radiative transfer equations and three profiles of fluxes and cooling rate obtained again.

[17] 6. Two pairs of the “new” profiles ( $F_U(Z)$  and  $Q(Z)$  in our example) should be compared against the line-by-line calculations (the third pair of them,  $F_D$ , should be identical). If differences are too great or too small (in comparison with the accuracy needed), threshold  $S^1$  should be decreased or increased, respectively, to narrow down or expand subset  $S^1$ . After that the procedure should be renewed from step 2 until the threshold and subset are found, which would ensure the required accuracy over the widest subset  $U^1$  possible.

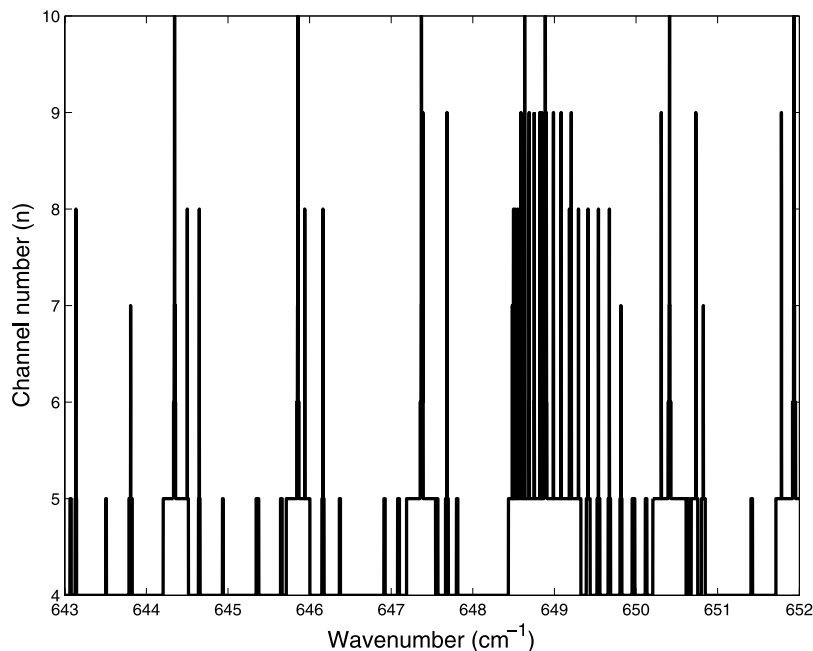
[18] 7. Then, the points included into  $U^1$  should be eliminated from further operations and the work should be started again from step 2 (another level  $Z^*$  can be used).

[19] The result is nearly independent of the  $Z^*$  level. Nevertheless, it can be recommended to take  $Z^*$  at the altitude where the cooling rate is at its maximum for the given channel. All this would allow one to find step-by-step all the subsets ( $U^1, U^2, \dots, U^N$ ), which will generate a  $k$ -distribution model, as well as “representative” absorption profiles  $K^1(Z), K^2(Z), \dots, K^N(Z)$  and representative cross sections  $\bar{K}^1(Z), \bar{K}^2(Z), \dots, \bar{K}^N(Z)$ . We will refer to these subsets, after the paper by *Nakajima et al.* [2000], as “channels.” An example of a system of channels for a portion of the  $15\text{-}\mu\text{m}$   $\text{CO}_2$  band is shown in Figure 2. The number at each wave number point indicates the channel number where this point should be placed. This case was chosen for illustration since it is the most complex in the longwave approximation (it needs 10 channels to be used here, as will be explained in the next chapter).

[20] To get a better insight into how the technique works, let us compare Figure 1 against Figure 3, where a four-term approximation is used. As it can be seen from Figure 3, the 4-term approximation error is  $<0.005\text{ Kd}^{-1}$  for cooling rate calculations, which is  $\sim 1\%$  of the one-term error. It is notable that only one term (namely, the fourth one) gives an accurate description of cooling rates in the whole stratosphere. As will be shown below, it is not a special case since the method has proven most efficient for stratosphere and upper atmosphere.

[21] Now let us consider overlapping absorption by different species. There are two ways to solve the problem of overlapping bands using the above technique. The simplest way consists of two stages.

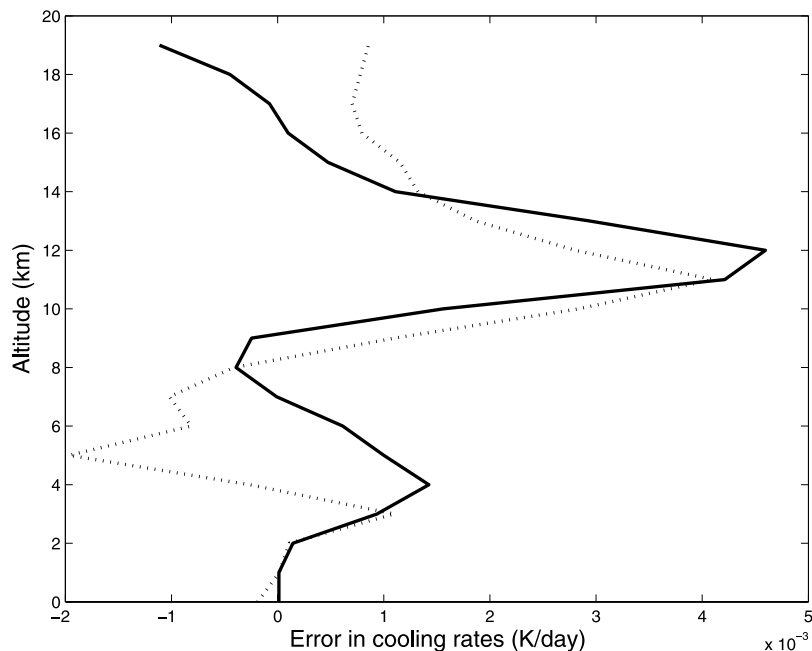
[22] 1. The technique is applied independently to two (or more) absorber species, which we shall denote as subscription 1 and 2. The idea is to obtain not only the



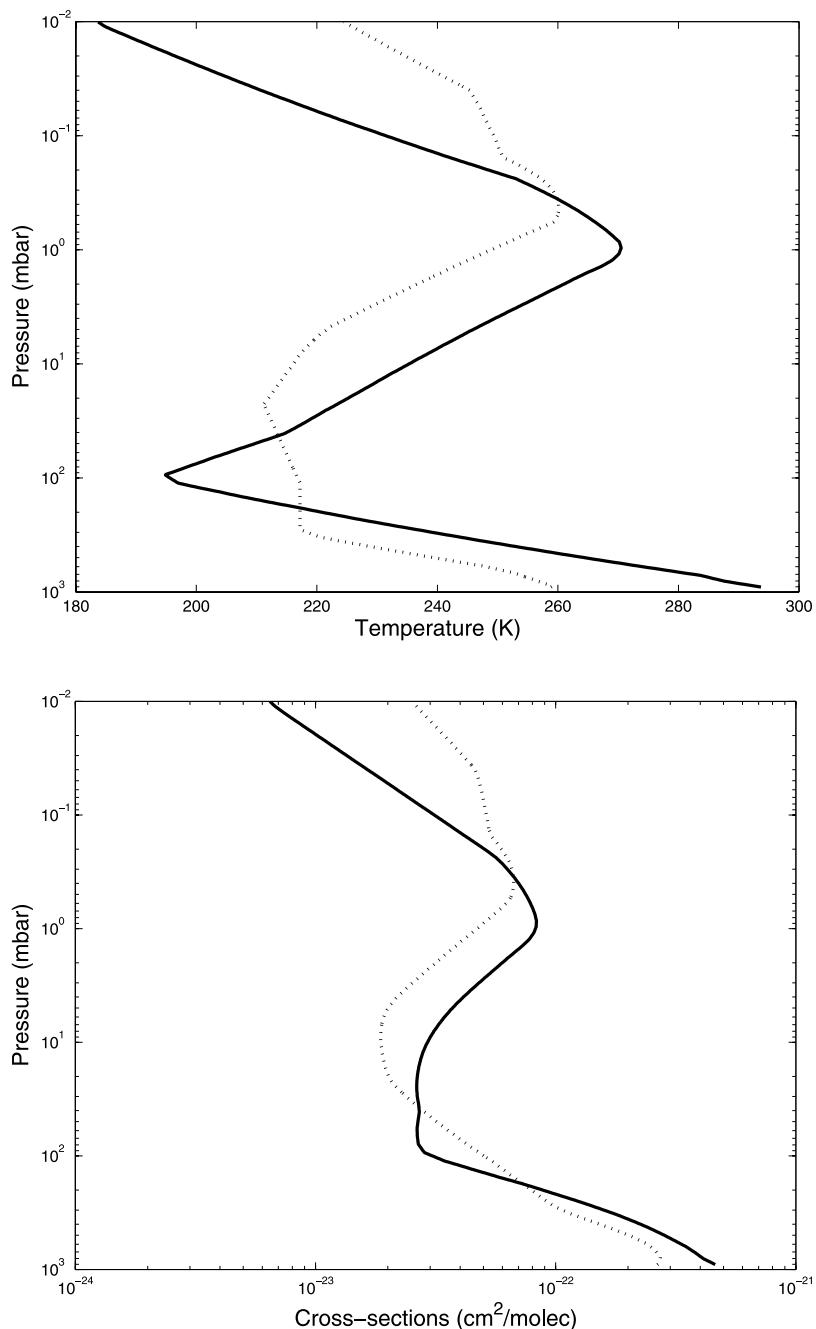
**Figure 2.** System of channels for a portion of the 15- $\mu\text{m}$   $\text{CO}_2$  band. The number at each wave number point indicates the number of the channel where this point should be placed.

profiles of representative absorption cross-sections  $\bar{K}_1^1(Z)$ ,  $\bar{K}_1^2(Z), \dots, \bar{K}_1^L(Z), \dots, \bar{K}_1^L(Z)$  and  $\bar{K}_2^1(Z), \bar{K}_2^2(Z), \dots, \bar{K}_2^M(Z), \dots, \bar{K}_2^M(Z)$  of k-terms for each species but also both systems of channels  $U_1^1, U_1^2, \dots, U_1^L, \dots, U_1^L$  and  $U_2^1, U_2^2, \dots, U_2^M, \dots, U_2^M$ , where L and M are the numbers of terms (or channels) for each species. In practice we have a pair of two-dimensional arrays where one dimension is for wave number points and the other is for channel numbers as shown in Figure 2. (A wave number grid must be obtained for the above line-by-line calculations.)

[23] 2. By means of comparison of these arrays (point after point), the number of common wave number points  $N_{IJ}$ , which are included simultaneously in  $U_1^I$  and  $U_2^J$ , should be counted for each combination of the I-th and J-th channels. Generally, there are  $L \times M$  combinations. Now we can easily define the parameters for all the  $L \times M$  terms of the mixture. It is evident that the IJ-th term  $U_{1+2}^{IJ} = U_1^I \cap U_2^J$  has representative absorption  $K_{1+2}^{IJ}(Z) = \bar{K}_1^I(Z) * \eta_1(Z) + \bar{K}_2^J(Z) * \eta_2(Z)$  and weight  $W_{IJ} = N_{IJ}/N_{\text{total}}$ , where  $N_{\text{total}}$  is the total number of points for the given spectral region and  $\eta_1, \eta_2$  are the molecular densities.



**Figure 3.** Errors in cooling rates. Same as in Figure 1 but for the case of one-term (dotted line) and four-term approximation (solid line). Errors for one-term approximation have been multiplied by factor 0.01.



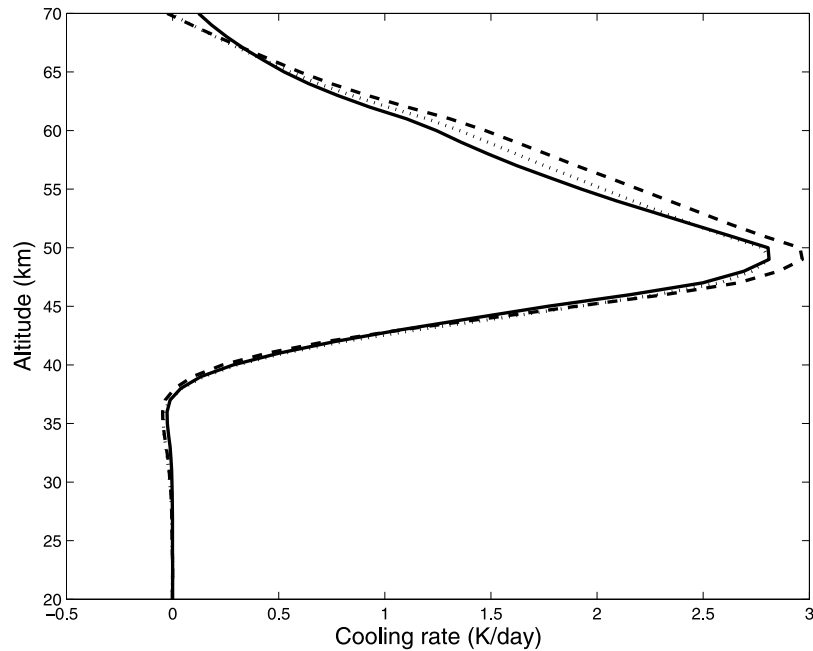
**Figure 4.** Temperatures  $T_{TRP}(P)$  (solid line) and  $T_{SAW}(P)$  (dotted line) and  $CO_2$  representative cross sections  $\bar{K}_{TRP}(P)$  (solid line) and  $\bar{K}_{SAW}(P)$  (dotted line) as a function of pressure  $P$  for Tropical (TRP) and Sub-Arctic Winter (SAW) atmospheres in the  $550\text{--}990\text{ cm}^{-1}$  spectral region.

[24] Unfortunately, this method where all the species are considered to be equivalent usually creates too many channels. Hence it has been applied directly to a few cases only (e.g., for halocarbons plus carbon dioxide). For other cases it has been used as a preliminary stage to select the channels where both (or more) species should be taken into account simultaneously. (In these channels both absorptions are comparable and the corresponding weights  $W_{IJ}$  are fairly large.) Then the other approach has been used.

[25] This approach (more effective but more complicated) is to define a system of channels for the second “minor”

species in the channels of the first “key” (most important) species. With this aim in view, it is necessary, firstly, to define a system of channels for the “key” species (the above mentioned array), and then (considering channel by channel separately) to apply the above procedure 1–7 only to the wave number points belonging to the channel under consideration. The absorption should be defined as above. The second approach is more effective than the first one since it offers a possibility to get directly the essential terms of the mixture taking into account information that absorption of the “key” specie is dominated. More details of the





**Figure 5.** Cooling rates for the doubled concentration  $\text{CO}_2$  in the sixth channel of band B (see Table 1) calculated by the line-by-line (solid line) and approximate methods with (dotted line) and without (dashed line) correction in tropical atmosphere.

technique suggested are described in the next section, where FKDM is considered.

### 3. Fast k-Distribution Model for the Longwave

[26] The desired accuracy is a major constraint upon any new radiation parameterization. The accuracy of Rapid Radiation Transfer Model (RRTM) [Mlawer *et al.*, 1997], which is one of the most precise at the moment, can be used as a reference. This accuracy is as follows: an error of less than  $1.0 \text{ W m}^{-2}$  for total net flux;  $0.07 \text{ K d}^{-1}$  for the total cooling rate error in troposphere and  $0.75 \text{ K day}^{-1}$  in stratosphere and higher. An error here is defined as the difference between line-by-line and approximate calculations. Unfortunately, line-by-line calculations are not absolutely exact due to some internal assumptions (in wave number and vertical integration, line cut off, etc.) on the one hand and uncertainties in the initial spectroscopic information on the other. Because of these assumptions, the intercomparison of different line-by-line calculations of downward surface radiance for the SPECTRE experiment have revealed discrepancies of  $\sim 1\%$  [Ellingson and Wiscombe, 1996]. Thus an accuracy of  $\sim 1\%$  or  $\sim 3\text{--}4 \text{ W m}^{-2}$  in upward and downward flux calculations and  $\sim 0.2 \text{ K/day}$  and  $1.0 \text{ K/day}$  in tropospheric and stratospheric calculations respectively may be regarded as acceptable [Cusack *et al.*, 1999].

[27] The technique, discussed in section 2, gives a possibility to get k-distributions with the required accuracy for any atmospheric model but at the expense of a rather time-consuming iterative procedure using line-by-line calculations. Below we will describe a simple fast algorithm for calculations of absorption coefficients for a variety of atmospheric profiles, which needs no preliminary line-by-line calculations. Figure 4 illustrates the temperature and

pressure dependence of a typical channel. It shows temperatures  $T_{\text{TRP}}(P)$  and  $T_{\text{SAW}}(P)$  (in K) and  $\text{CO}_2$  representative cross-sections  $\bar{K}_{\text{TRP}}(P)$  and  $\bar{K}_{\text{SAW}}(P)$  (in  $\text{cm}^2 \text{ molecule}^{-1}$ ) as a function of pressure  $P$  (in mbar) for Tropical (TRP) and Sub-Arctic Winter (SAW) atmospheres. The cross-section profiles have been obtained from line-by-line downward fluxes in the  $550\text{--}990 \text{ cm}^{-1}$  region for the third channel (moderate absorption). These plots show a clear correlation between temperature and the channel absorption coefficient, which is caused by the temperature dependence of the line-

**Table 1.** Band and Channel Allocation

| Band | Limit, $\text{cm}^{-1}$ | Channels | Species in Channels  |
|------|-------------------------|----------|--|
| A    | 40–550                  | 4        | $\text{H}_2\text{O}$ only  |
| B    | 550–990                 | 10       | 1- $\text{H}_2\text{O}$ , $\text{CO}_2$ , $\text{O}_3$ ,<br>CFC-11, CFC-12   |
|      |                         |          | 2- $\text{H}_2\text{O}$ , $\text{CO}_2$ , $\text{N}_2\text{O}$<br>3,4- $\text{CO}_2$ , $\text{H}_2\text{O}$ , $\text{O}_3$ , $\text{N}_2\text{O}$<br>5- $\text{CO}_2$ , $\text{H}_2\text{O}$ , $\text{O}_3$<br>6, 7, 8- $\text{CO}_2$ , $\text{H}_2\text{O}$<br>9, 10- $\text{CO}_2$ |
| C    | 990–1400                | 6        | 1- $\text{H}_2\text{O}$ , $\text{CO}_2$ , $\text{O}_3$ ,<br>$\text{N}_2\text{O}$ , $\text{CH}_4$ , CFC-11,<br>CFC-12, CFC-113  |
|      |                         |          | 2- $\text{H}_2\text{O}$ , $\text{CO}_2$ , $\text{N}_2\text{O}$ ,<br>$\text{CH}_4$ , CFC-11, CFC-12,<br>CFC-113   |
|      |                         |          | 3- $\text{O}_3$ , $\text{H}_2\text{O}$ , $\text{CO}_2$ ,<br>CFC-11, CFC-12,<br>CFC-13  |
|      |                         |          | 4, 5- $\text{O}_3$ , $\text{H}_2\text{O}$ , $\text{CO}_2$<br>6- $\text{O}_3$ , $\text{H}_2\text{O}$  |
| D    | 1400–3000               | 3        | 1- $\text{H}_2\text{O}$ , $\text{CO}_2$<br>2, 3- $\text{H}_2\text{O}$  |
|      |                         |          | Total  |

**Table 2.** Comparison of Correlated-k Model and the Model Under Discussion<sup>a</sup>

| Reference                        | Bands | Terms | $\Delta_T$ | $\Delta_S$ |
|----------------------------------|-------|-------|------------|------------|
| [ <i>Mlawer et al.</i> , 1997]   | 16    | 256   | 0.07       | 0.75       |
| [ <i>Cusack et al.</i> , 1999]   | 8     | 33    | 0.2        | 1.5        |
| [ <i>Nakajima et al.</i> , 2000] | 9     | 40    | 0.15       | 0.5        |
| This work                        | 4     | 23    | 0.15       | 0.9        |

<sup>a</sup>Standard atmospheres are used only.  $\Delta_T$  and  $\Delta_S$  are cooling rate errors in troposphere and stratosphere (in K day<sup>-1</sup>).

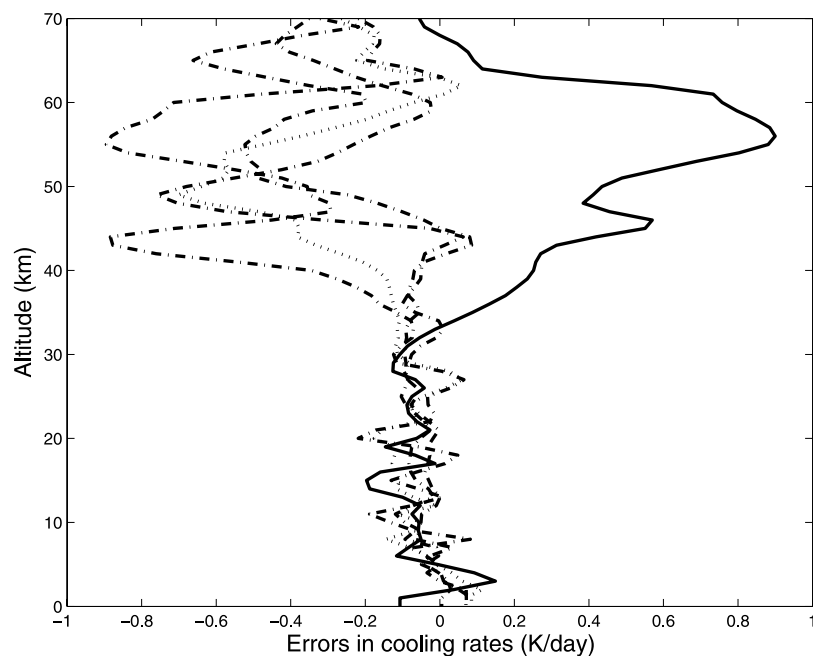
by-line absorption coefficients, thus indicating that the absorption coefficient for this channel reflects the real behavior of the spectrum that it represents. As a result, for this version of FKDM we have chosen the simplest algorithm making use of a linear temperature interpolation:

$$K(P, T) = [K_{TRP}(P) * (T - T_{SAW}(P)) + \bar{K}_{SAW}(P) * (T_{TRP}(P) - T)] * \eta(P) / (T_{TRP}(P) - T_{SAW}(P)), \quad (1)$$

where  $\eta$  is the molecular density in molecule/(cm<sup>2</sup>km), thus yielding  $K(P, T)$  in km<sup>-1</sup>. Node points in each channel for the given pressure,  $P$ , are the above-mentioned values ( $T_{TRP}(P)$ ,  $\bar{K}_{TRP}(P)$ ) and ( $T_{SAW}(P)$ ,  $\bar{K}_{SAW}(P)$ ). It should be mentioned that a common method to account for absorption coefficient variations with pressure and temperature is to scale a reference  $K$  value by an appropriate function of pressure and temperature [e.g., *Cusack et al.*, 1999]. But it was more convenient to use equation (1) in FKDM instead of scaling approximation due to the fact that profiles of representative absorption coefficients sometimes reveal a rather complex behavior [*Fomin*, 2003]. Moreover, equation (1) usually needs fewer calculations than scaling approximation (linear

interpolation is utilized also in the work of *Mlawer et al.* [1997]).

[28] Equation (1) gives acceptable accuracy in flux and cooling rate calculations for all the standard atmospheres. FKDM's accuracy was assessed for standard atmospheres with different concentrations of greenhouse gases and for a set of real atmospheric conditions (see the next section for details). It has been found that for cases with variable concentration of greenhouse gases the two reference profiles give flux sensitivities with insufficient accuracy. The straightforward way is to use a set of profiles corresponding to concentrations in equation (1), but an alternative method has been devised to solve this problem. The point is that representative cross sections of some species obtained for a given concentration, can be slightly corrected for another concentration, e.g., doubled. Figure 5 illustrates this statement. This Figure shows cooling rates by CO<sub>2</sub> in the sixth channel of band B (see Table 1) for the doubled concentration in the tropical atmosphere. These cooling rates were calculated by the line-by-line and approximate methods with and without correction. Correction in this case means multiplying the initial cross-section, obtained for the present CO<sub>2</sub> concentration, by the scalar factor  $\sim 0.9$ . As has been found, the factor for other channels is equal to 0.8–0.9 and 1.1–1.2 for doubled and halved concentrations, respectively. In the current version of FKDM for other concentrations this factor is defined using simple parabolic interpolations, where halved, normal and doubled concentrations are used as node points. A similar correction has been introduced for H<sub>2</sub>O also because calculation errors slightly exceeded an acceptable accuracy in very dry atmospheres. It should be stressed that all these scalar factors can be defined for any atmospheric model from amounts of species before flux and cooling rate calculations. (For H<sub>2</sub>O the scalar factor is a



**Figure 6.** Cooling rate error profiles for standard tropical (solid line), midlatitude summer (dashed line), midlatitude winter (dotted line), subarctic summer (dashed-dotted line) and subarctic winter (dashed-double dotted line) atmospheres.

**Table 3.** Nadir Downward Radiances ( $\text{W m}^{-2} \text{sr}^{-1}$ ) at the Surface From Measurements Taken for Four Atmospheres in the SPECTRE Campaign<sup>a</sup>

|                             | Case 1 | Case 2 | Case 3 | Case 4 |
|-----------------------------|--------|--------|--------|--------|
| Surface temperature, K      | 290.9  | 283.3  | 272.8  | 275.7  |
| H <sub>2</sub> O amount, cm | 1.77   | 0.55   | 0.73   | 0.95   |
| SPECTRE                     | 40.215 | 29.768 | 24.094 | 28.462 |
| LBLRTM-SPECTRE              | -0.154 | 0.069  | -0.328 | -0.525 |
| FLBLM-SPECTRE               | 0.278  | 0.284  | -0.198 | -0.306 |
| FKDM- FLBLM                 | -0.749 | 0.258  | -0.571 | -0.231 |

<sup>a</sup>Spectral range: 630–2600  $\text{cm}^{-1}$ . Differences between the measurements and calculation results obtained with LBLRTM, FLBLM, and FKDM.

function of the water vapor amount.) This correction holds the resulting errors within the acceptable accuracy for all the cases considered for the validation and this simple algorithm was accepted for the current version of FKDM. It should be mentioned that each profile, as shown in Figure 4, can be approximated by interpolation using a dozen of points only (in the work of *Fomin* [2003], 15 node points are used). Thus it is not a particular problem to use, on necessity, many more than just a pair of profiles for interpolation to double the present model accuracy without increasing the number of channels and, consequently, without decreasing the calculation speed. This fact may also be useful for other applications of the technique (e.g., in satellite investigations) if a necessity arises to reach high accuracy by means of using a wide set of reference profiles.

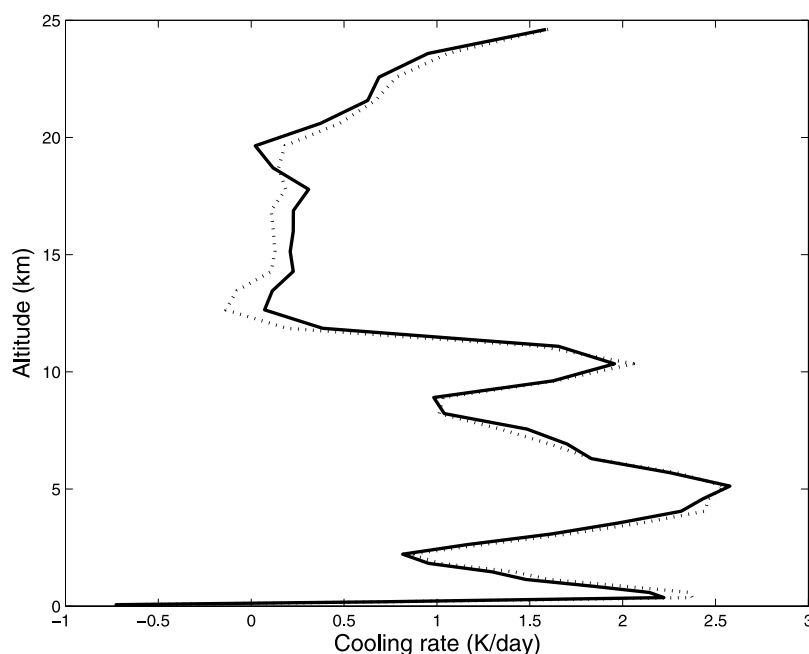
[29] To take into account thermal emission, a common method uses the Planck function spectrally integrated over each band, usually fitted by temperature polynomial. Then thermal emission in each channel of the given band is defined as multiplication of integrated Planck function by the channel's weight. In a wide-band case and if spectral

regions with weak and strong absorption are distributed inhomogeneously inside the band it may lead to some errors in further calculations. Alternatively, the Planck functions for FKDM were integrated over all channels  $U^1, U^2, \dots$  and stored as tables with the temperature step of 0.1 K. The tables are used for linear interpolation, which is faster than polynomial fitting. Other parameters such as atmospheric scattering properties can be defined in a similar fashion.

[30] Combined bands from the code suggested by *Nakajima et al.* [2000] have been used for the given model (see Table 1). In Table 1, the first-named gas is the key species and has been used to define the wave number subset of this channel and is therefore more accurately simulated than any other overlapping gas.

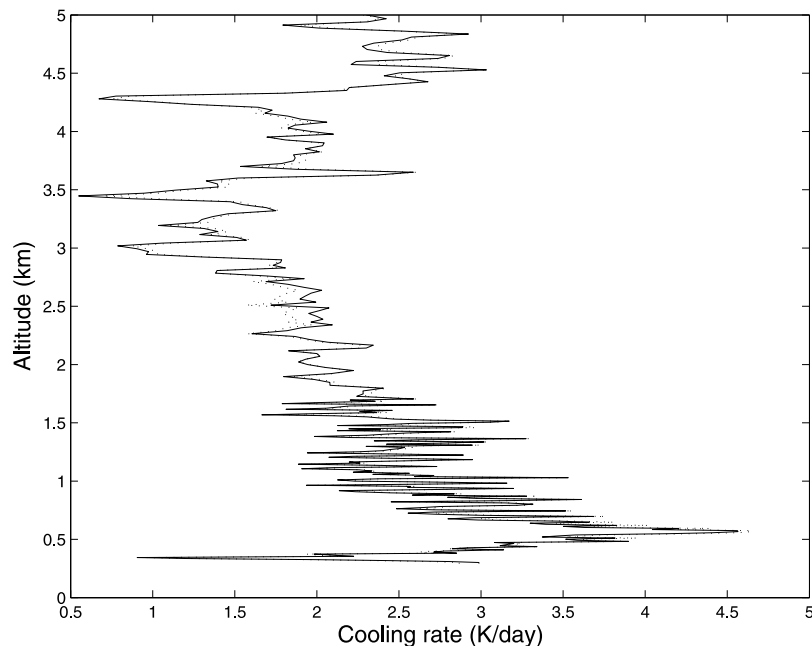
[31] The foreign-broadened continuum of H<sub>2</sub>O is included with the line absorption. The self-broadened continuum is taken into account in the B, C, and D bands as a gray absorber, using the analytical fit borrowed directly from the CKD model.

[32] The cooling rate errors for standard tropical (TRP), midlatitude summer (MLS), midlatitude winter (MLW), subarctic summer (SAS), and subarctic winter (SAW) atmospheres are within  $\Delta_T = 0.15 \text{ K day}^{-1}$  and  $\Delta_S = 0.9 \text{ K day}^{-1}$  in troposphere and stratosphere (up to 70 km), respectively (see section 4). It should also be mentioned that errors for upward and downward fluxes are usually within 1–2  $\text{W m}^{-2}$  and always within  $\sim 4 \text{ W m}^{-2}$  for the same cases. Table 2 permits for a comparison of the data of the model under consideration and other k-distribution models. As can be seen from Table 2, all models have comparable accuracy, but the discussed model has by  $\sim 1.5$ , 2, and 10 times fewer k-distribution terms than the rest of the models. Also note that 5 to 7 terms are sufficient to describe atmospheric radiation above  $\sim 40 \text{ km}$  in the model. These terms are related to very narrow CO<sub>2</sub> and O<sub>3</sub> bands, where



**Figure 7.** Line-by-line (solid line) and FKDM (dotted line) cooling rates for SPECTRE case 4.





**Figure 8.** Line-by-line (solid line) and FKDM (dotted line) cooling rates for the balloon-borne temperature and humidity sounding in tropical atmosphere.

fluxes are equal to approximately  $0.1 \text{ W m}^{-2}$  and less. Hence only  $\sim 15$  channels are needed for radiation calculations below  $\sim 40$  km.

#### 4. Validations

[33] The latest versions of spectral database HITRAN-11v [Rothman *et al.*, 2003] and water vapor continuum model CKD-2.4 [Mlawer *et al.*, 1999] have been used in FLBLM, based on the same physical assumptions used in the well known and carefully validated the Line-by-Line Radiation Transfer Model (LBLRTM) [Clough *et al.*, 1992; Clough and Iacono, 1995]. However, the two line-by-line models make use of different algorithms for gas absorption calculations and vertical integration. The first algorithm [Fomin, 1994] was independently tested [Kuntz and Hofner, 1999] and the properties of the second one based on the “linear in  $\tau$ ” approximation [Wiscombe, 1976] for vertically inhomogeneous atmosphere are well known. It is also worthy of note that a large family of calculations obtained with earlier versions of FLBLM was published long ago as benchmark calculations for the validation of radiation codes being used in climate models [Feigelson *et al.*, 1991; Fomin and Gershanov, 1997]. Of course these calculations have been used for FKDM validation. Figure 6 shows cooling rate error profiles for standard tropical (TRP), midlatitude

summer (MLS), midlatitude winter (MLW), subarctic summer (SAS), and subarctic winter (SAW) atmospheres. FLBLM has also been validated by the ICRCCM working group using the SPECTRE field experiment [Ellingson and Wiscombe, 1996]. This experiment can be used for FKDM validation as well.

##### 4.1. Real Atmospheres

[34] It was necessary to confirm that “representative” absorption coefficients obtained above with artificial smooth atmospheric profiles (TRP and SAW) are applicable for radiative transfer simulation in real atmospheres. Table 3 presents the results of downward surface radiance calculations in the  $630\text{--}2600 \text{ cm}^{-1}$  spectral region for four profiles from the SPECTRE campaign. Note that these profiles have a rather complex structure. Three of the four profiles even involve temperature inversion. It is important for validation that these profiles differ both in the water vapor column ranging from  $0.55 \text{ cm}$  to  $1.77 \text{ cm}$  and in surface temperature varying from  $272.8 \text{ K}$  to  $290.9 \text{ K}$ .

[35] Table 3 also presents calculations with LBLRTM [Mlawer *et al.*, 1997], where HITRAN-92 and the CKD-2.1 model have been used. The data in Table 3 indicate that the differences between the two methods of line-by-line calculations and measurements are approximately the same and make less than 1.8% and 1.1% for the LBLRTM and

**Table 4.** Fluxes Perturbations at the Surface, Tropopause, and TOA for MLS Atmosphere Due to Doubling the  $\text{CO}_2$ ,  $\text{N}_2\text{O}$ ,  $\text{CH}_4$  Concentrations and Addition of Halocarbons (From 0.0 to 250, 500, and 100 pptv for CFC-11, CFC-12, and CFC-113)

|                      | Surface Down |      | Up    |       | Down  |      | Net   |       | TOA Up |       |
|----------------------|--------------|------|-------|-------|-------|------|-------|-------|--------|-------|
|                      | FLBLM        | FKDM | FLBLM | FKDM  | FLBLM | FKDM | FLBLM | FKDM  | FLBLM  | FKDM  |
| $\text{CO}_2$        | 1.70         | 1.93 | -3.80 | -3.97 | 1.66  | 1.66 | -5.46 | -5.63 | -2.67  | -2.57 |
| $\text{N}_2\text{O}$ | 0.35         | 0.55 | -0.89 | -0.92 | 0.17  | 0.1  | -1.06 | -1.08 | -0.99  | -1.17 |
| $\text{CH}_4$        | 0.57         | 0.24 | -0.66 | -0.66 | 0.07  | 0.08 | -0.73 | -0.74 | -0.69  | -0.87 |
| CFC                  | 0.28         | 0.26 | -0.29 | -0.29 | 0.05  | 0.05 | -0.34 | -0.34 | -0.43  | -0.43 |

FLBLM models, respectively. The differences between the line-by-line calculations are within  $\sim 1\%$ , which can be attributed to different versions of the HITRAN database and the CKD model [Fomin *et al.*, 2004]. The differences between FLBLM and FKDM calculations are within 2.4%, which is slightly greater than the maximum difference between line-by-line calculations and the experimental data. It should be stressed that the agreement between the FLBLM and FKDM calculations will improve if downward fluxes at the surface over the whole  $50\text{--}3000\text{ cm}^{-1}$  spectral region are taken into account. For this spectral region discrepancies in flux calculations at the surface and other horizontal levels have been found to be within  $\sim 1.3$  and  $\sim 3\text{ W m}^{-2}$  or  $\sim 0.5\%$  and  $\sim 1\%$ , respectively.

[36] Figure 7 shows the FLBLM and FKDM cooling rates for SPECTRE case 4, where the maximum discrepancies have been found. Nevertheless, even in this case the differences are less than  $\sim 0.2\text{ K day}^{-1}$ .

[37] In addition to the SPECTRE data, which have been obtained under conditions of comparatively dry atmosphere, a case of an observed tropical atmosphere has been used for validation (Amazon region, September 2002). The temperature and humidity profiles in this case have been taken directly from balloon borne soundings (water vapor column of 3.57 cm, surface temperature 298.5 K,  $\sim 900$  points over the range from 292 m to  $\sim 30$  km). Figure 8 shows FLBLM and FKDM cooling rates for this case. As can be seen, the discrepancies are below 0.2 K even for these strongly inhomogeneous conditions. The FLBLM and FKDM values for downward fluxes at the surface are  $378.9\text{ W m}^{-2}$  and  $377.0\text{ W m}^{-2}$ , respectively.

#### 4.2. Doubled $\text{CO}_2$ , $\text{N}_2\text{O}$ , and $\text{CH}_4$ Concentrations and Additions of Halocarbons

[38] To validate the FKDM sensitivity to variations of  $\text{CO}_2$ ,  $\text{N}_2\text{O}$ ,  $\text{CH}_4$ , and halocarbon concentrations, both FLBLM and FKDM have been used to calculate the differences in fluxes for the MLS atmosphere resulting from doubling the concentrations of the first three gases as compared to their current levels and from addition of halocarbons (from 0.0 to 250, 500, and 100 pptv for CFC-11, CFC-12, and CFC-113, respectively). These differences at the surface, in the tropopause (13 km) and at the top of atmosphere (TOA) are presented in Table 4. The values given in Table 4 indicate that FKDM provides approximately the same results as FLBLM does for doubled  $\text{CO}_2$  concentration and halocarbons additions, with errors of less than  $\sim 10\%$ . The results for  $\text{N}_2\text{O}$  and  $\text{CH}_4$ , regarded in this model version as “minor species” only, reveal larger relative errors. Nevertheless, the error of FKDM in simulating the radiative forcing (a difference in net fluxes at the tropopause) [Houghton, 1995], which is one of the most important parameters for climate investigations, was below  $\sim 3\%$  for all the cases considered in Table 4. Also note that the corresponding differences in cooling rates have been found to be negligible (less than  $0.1\text{ K day}^{-1}$ ).

### 5. Summary

[39] A new k distribution method suitable for fast radiative transfer codes has been described, and an assessment of its accuracy and speed has shown it to be a more efficient

technique. This improvement is largely due to a direct use of line-by-line fluxes and cooling rates for the determination of the values of the k terms. The method permits a more accurate treatment of overlapping absorbers in a spectral region, which also contributes to its improved efficiency. An empirical scaling factor is applied to k terms for  $\text{H}_2\text{O}$  and  $\text{CO}_2$ , in order to achieve the desired accuracy in atmospheres with extreme absorber amounts, and this also contributes to the improved efficiency of this method.

[40] The assessment of FKDM, a new longwave parametrization based on the k-distribution method, has shown clearly that it is suitable for weather and climate prediction, as well as for flux and cooling rate calculations in GCM experiments. FKDM offers the best compromise between speed of execution and accuracy for any published longwave parametrization based on the k-distribution method. It is almost twice as fast as similar methods, yet maintains comparable levels of accuracy for fluxes and cooling rates. The net flux perturbations calculated by FKDM caused by realistic changes in greenhouse gas concentrations have an error of similar magnitude to those produced by other climate model radiation schemes. It should be stressed that the errors in the stratosphere are similar to those in the troposphere. So this new method also offers a significant improvement in efficiency for wide-band satellite data processing methods.

[41] It should also be mentioned that this version of FKDM is not at its highest achievable accuracy. It is anticipated that future developments in the specification of the temperature dependence of k values will improve results. This work needs additional effort and is planned for the next version of FKDM. Moreover, we intend to take into account the scattering processes in FKDM or to adapt it for the existing radiation block of climate models: its similarities with other k-distribution models suggests it should have a similar performance in this regards. It is planned to extend FKDM in the vertical from 70 to 120–150 km (using NLTE theory) to make use of its good performance at high altitudes. However, at first, our intention is to extend FKDM to the shortwave spectral region.

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B. A. Fomin, CPTEC/INPE, Cachoeira Paulista, Sao Paulo, 12630-000, Brazil. (fomin@cptec.inpe.br)